# Preface

This book attempts to give students a unified introduction to the models, methods, and theory of modern linear algebra. Linear models are now used at least as widely as calculus-based models. The world today is commonly thought to consist of large, complex systems with many input and output variables. Linear models are the primary tool for analyzing these systems. A course based on this book (or one like it) should prove to be the most useful college mathematics course most students ever take. With this goal in mind, the material is presented with an eye toward making it easy to remember, not just for the next hour test but for a lifetime of diverse uses.

Linear algebra is an ideal subject for a lower-level college course in mathematics, because the theory, numerical techniques, and applications are interwoven so beautifully. The theory of linear algebra is powerful, yet easily accessible. Best of all, theory in linear algebra is likable. It simplifies and clarifies the workings of linear models and related computations. This is what mathematics is really about, making things simple and clear. It provides important answers that go beyond results we could obtain by brute computation. For too many students, mathematics is either a collection of techniques, as in calculus, or a collection of formal theory with limited applications, as in most courses after calculus (including traditional linear algebra courses). This book tries to rectify this artificial dichotomy.

Again, the applications of linear algebra are powerful, easily understood, and very diverse. This book introduces students to economic input–output models, population growth models, Markov chains, linear programming, computer graphics, regression and other statistical techniques, numerical methods for approximate solutions to most calculus problems, linear codes, and much more. These different applications reinforce each other and associated theory. Indeed, without these motivating applications, several of the more theoretical topics could not be covered in an introductory textbook.

The field of linear numerical analysis is very young, having been dependent on digital computers for its development. This field has wrought major changes in what linear algebra theory should be taught in an introductory course. The standout example of such a modern linear algebra text is G. Strang's *Linear Algebra and Its Applications*. Once the theory was needed as an alternative to numerical computation, which was hopelessly difficult. Now theory helps direct and interpret the numerical computation, which computers do for us.

*Overview of the Text* This book develops linear algebra around matrices. Vector spaces in the abstract are not considered, only vector spaces associated with matrices. This book puts problem solving and an intuitive treatment of theory first, with a proof-oriented approach intended to come in a second course, the same way that calculus is taught.

The book's organization is straightforward: Chapter 1 has introductory linear models; Chapter 2 has the basics of matrix algebra; Chapter 3 develops different ways to solve a system of equations; Chapter 4 has applications, and Chapter 5 has vector-space theory associated with matrices and related topics such as pseudoinverses and orthogonalization. Many linear algebra textbooks start immediately with Gaussian elimination, before any matrix algebra. Here we first pose problems in Chapter 1, then develop a mathematical language for representing and recasting the problems in Chapter 2, and then look at ways to solve the problems in Chapter 3—four different solution methods are presented with an analysis of strengths and weaknesses of each.

In most applications of linear algebra, the most difficult aspect is understanding matrix expressions, such as $\mathbf{U}e^{\mathbf{D}}\mathbf{U}^{-1}$. Students from a traditional linear algebra course have little preparation for understanding such expressions. This book constantly forces students to interpret the meaning of matrix expressions, not just perform rote computations. Matrix notation is used as much as possible, rather than constantly writing out systems of equations. The sections are generally too long to be covered completely in class; most have several examples (based on familiar models) that are designed to be read by students on their own without explanation by the instructor. The goal is for students to be able to read and understand uses of matrix algebra for themselves.

The material is unified pedagogically by the repeated use of a few linear models to illustrate all new concepts and techniques. These models give the student mental pictures to "visualize" new ideas during this course and help remember the ideas after the course is over.

Although this book is often informal ("proving theorems" by example) and sticks mainly to matrices rather than general linear transformations, it covers several topics normally left to a more advanced course, such as matrix norms, matrix decompositions, and approximation by orthogonal polynomials. These advanced topics find immediate, concrete applications. In addition, they are finite-dimensional versions of important theory in functional analysis; for example, the eigenvalue decomposition of a matrix into simple matrices is a special case of the spectral representation of linear operators.

*Discrete Versus Continuous Mathematics* Today there is a major curriculum debate in the mathematics community between computer science–oriented discrete mathematics and classical calculus-based mathematics. Linear algebra, especially as viewed in this book, is right in the middle of this debate. (Linear algebra and matrices have always been in the middle of such debates. Matrices were a core topic in the best-known first-year college mathematics text before 1950, Hall and Knight's *College Algebra;* and much of Kemeny, Snell, and Thompson's *Introduction to Finite Mathematics* involved new applications of linear algebra: Markov chains and linear programming.)

This book attempts to present a healthy interplay between mathematics and computer science, that is, between continuous and discrete modes of thinking. The complementary roles of continuous and discrete thinking are typified by the different uses of the euclidean norm ($l_2$-norm) and sum norm ($l_1$-norm) in this book. An important example of computer science thinking in this book is matrix representations, such as the **LU** decomposition. They are viewed as a way to preprocess the data in a matrix in order to be ready to solve quickly certain types of matrix problems.

We note that computer science even gives insights into the teaching of any linear algebra course. A computer scientist's distinction between high-level languages (such as PASCAL) and low-level languages (such as assembly language) applies to linear algebra proofs: A high-level proof involves matrix notation, such as $\mathbf{B}^T\mathbf{A}^T = (\mathbf{AB})^T$, while a low-level proof involves individual entries $a_{ij}$, such as $c_{ij} = \Sigma a_{ik}b_{kj}$.

*Suggested Course Syllabus* This book contains more than can be covered in the typical first-semester sophomore course for which it is intended. Most of Chapters 1, 2, and 3 and the first four sections of Chapter 5 should normally be covered. A freshman course would skip Chapter 5. In addition, selected sections of Chapter 4 can be chosen based on available time and the class's interests. For the student, the essence of any course should be the homework. This book has a large number of exercises at all levels of difficulty: computational exercises, applications, and proofs of much of the basic theory (with extensive hints for harder proofs). For more information about course outlines, plus suggested homework sets, sample exams, and additional solutions of exercises, see the accompanying Instructor's Manual.

At the end of the book is a list of various programming languages and software packages available for performing matrix operations. It is recommended that students have access to computers with ready matrix software in the first week.

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After the first draft was finished and initial reviews indicated that much rewriting was needed, the book might have died had not Don Small offered to help with the revision work. He added new examples, reworked several sections, and generally made the text better.

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A. T.