May 2020

recitation_IS19_20160330_Seg01.pdf

Setting: S19 leads a recitation section.

Participants: IS19 (boy, blue hooded sweatshirt), S1 (boy, black sweatshirt, backpack), S2 (boy, leather jacket), RA1 (gold hoop earrings)

(0:00)
XXX IS19: um
XXX ((pause))
(0:10)
XXX S1: ((approaches IS19))
XXX IS19: so this is
XXX S1: I have an exam in like an hour
XXX IS19: ok sure
XXX S1: so I'm just gonna take these and
XXX IS19: ok
XXX thank you
XXX S1: no problem
XXX IS19: good luck
XXX S1: thank you
XXX S1: ((leaves))
XXX S2: ((approaches IS19))
XXX just one question
XXX IS19: ((looks at the paper))
XXX S2: it's constant,
XXX IS19: yes it's constant
XXX S2: oh
XXX how is it decreasing?
XXX IS19: because it's a (coat) in
XXX (coat dollar s) form
XXX it must be constant
XXX S2: oh
XXX cause together equals
XXX k to the y,
XXX IS19: yeah
XXX S2: uh kl to the y,
XXX IS19: yeah
XXX because you put a z in over
XXX uh before k and
XXX (z 3) and you ((indistinguishable)) to put (z)
XXX to k and l
XXX S2: mhm
XXX IS19: and uh some of the power is y
XXX and the z
XXX so there is a z (I found out) this whole thing
XXX S2: mm ok
XXX IS19: ((nodding))
XXX S2: gotcha
XXX thank you
XXX ((leaves))
XXX RA1: ((whispers to IS19)) I have a quick announcement

XXX IS19: ok sure
XXX RA1: um as you guys know
XXX from the last class
XXX when we were here
XXX we’re doing a research project,
XXX and if you didn’t sign one of our consent forms
XXX then you can sign one now.
XXX ((nodding))
XXX did everybody sign it?
XXX ok
XXX if you didn’t just let me know
XXX IS19: ok so let’s start
XXX talking about this homework now and,
XXX I’ll concentrate on problem
XXX uh 3 and 4
XXX and if we have time
XXX then I’ll discuss about the first 2 questions
XXX so first m- let me to do a quick review
XXX about how we-
XXX the procedure for how we solve this kind of
XXX uh steady state or golden rule problem
XXX so.
XXX s-
XXX so typically you will
XXX be given a aggregate production function
XXX which in (.) this form
XXX ah °sorry
XXX ((erases))
XXX so it’s
XXX f to the aggregate capital
XXX and the efficient labor
XXX and
XXX always you need to first find out what is the
XXX prefect to worker production function
XXX uh sorry
XXX it’s- ((fixes on board))
XXX and here the small y equals to the capital Y
XXX the aggregate output
XXX divided by ((writes)) the efficient labor
XXX and the small k equals to
XXX the aggregate K divided by
XXX this uh total efficient labor
XXX and (.1)
XXX then you need to find out
XXX what is the (.1) all the le-
XXX uh all the number value of a (())
XXX in the steady state
XXX and to do this,
XXX first
XXX we need to find out ((writing))
XXX what is the value of k star
XXX and we use the condition of

XXX delta k↑ equals to zero
XXX and ((writing))
XXX as we know
XXX the- the k equals to ((writing))
XXX the investment
(3:00)
XXX minus (.1)
XXX the break even value
XXX which is n + g + delta times k the capital (.3)
XXX so
XXX and the investment is always equal to saving
XXX and the saving equals to the saving rate
XXX times the output
XXX or the production function. (.3)
XXX minus
XXX the same thing. (.3)
XXX and uh
XXX this is delta k
XXX delta k.
XXX and since we have the condition that
XXX delta k the changing capital is zero,
XXX so
XXX ((pause for writing))
XXX we will use this equation
XXX the saving equals to the break even level
XXX ((pause for writing))
XXX we use this equation to find what is the
XXX capital in steady state
XXX so this is k star here
XXX and k star here
XXX so this is
XXX always the- uh the first step
XXX to solve this problem
XXX so to find out
XXX what is the capital in steady state
XXX using this equation
XXX and i- in the second step
XXX with the value of the (.) k star
XXX we can find y star
XXX the uh output for efficient worker
XXX in steady state
XXX using the production function
XXX uh you have derived in- in the first part
XXX so this part
XXX from this equation you can get y star (.1)
XXX and then you can get I star because
XXX I star equals to n plus g plus (.) delta,
XXX times k star
XXX and this is from the up-
XXX again this condition
XXX with k equals to zero

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XXX        because this is uh the
XXX        and because of this equation I
XXX        should always be equal to this break even value
XXX        and from this equation
XXX        we can (.)
XXX        have what is the value of (.) I star
XXX        investment in steady state
XXX        a:nd finally we can find
XXX        consumption through the simple equation that
XXX        uh it equal to y star minus I star
XXX        we know the value of y star from step 2
XXX        and the value of I star from step 3
XXX        so from this equation we
XXX        can get the cap consumption steady state
XXX        so ((gesturing at board)) that is the-
XXX        the procedure we need to follow
XXX        to get- to solve the problem
XXX        and it is not the only way that you can do it
XXX        you can use some (.) other equation
XXX        for example for the investment
(6:00)
XXX        you can use
XXX        uh
XXX        I star equals to s times f k star
XXX        the investment equals to saving rate
XXX        but you can get the same result.
XXX        so I just ((gesturing at board))
XXX        show you (.) the procedure that I always do
XXX        to solve this problem
XXX        and this is for a steady state
XXX        and for the golden rule level
XXX        ((pause for writing))
XXX        again in the same uh- in the first step
XXX        we need to find out what is the value of
XXX        capital
XXX        we notate by k star g r
XXX        and the condition we use here is
XXX        m p k the marginal product in capital
XXX        equals to (.) m plus (.) g plus (.) delta
XXX        this uh is the
XXX        population growth rate
XXX        the g is the technology growth rate
XXX        and delta is the depreciation rate.
XXX        so you should be very clear about the difference
XXX        between this condition and ((pointing))
XXX        and that one for the steady state
XXX        here you need to times the k star
XXX        but ((pointing)) in this equation
XXX        there is no k star
XXX        the right hand side only (.) include this
XXX        three parameters (.2)
XXX        so
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XXX        from this equation
XXX        we have got the value of k star
XXX        because
XXX        these three parameters are known
XXX        and the mpk only depends on- uh o-on k
XXX        from this ((pointing))
XXX        from this production function
XXX        so from this equation we can
XXX        gather value of k star
XXX        in golden rule.
XXX        and ((writing))
XXX        the second step is the same
XXX        and using the production function,
XXX        we can find
XXX        the golden rule
XXX        um
XXX        level of uh output for efficient worker
XXX        so it equals to
XXX        f k star p
XXX        uh golden rule
XXX        so from this equation
XXX        we know the value of output
XXX        and the (.) third step and the fourth (.) step
XXX        are also the same
XXX        so I star
XXX        and plus g plus (.) the depreciation rate (.) times
XXX        k (.) star in golden rule
XXX        so we can get ((writing))
XXX        I star
XXX        and (.) again the consumption
XXX        equals to output ((writing))
XXX        minus investment
(9:01)
XXX        and we have one more step here
XXX        because we also need to find out what is the
XXX        golden rule saving rate
XXX        so (.2)
XXX        at uh
XXX        you can use two,- two approaches
XXX        so
XXX        the first condition is the consumption
XXX        always equals to uh (.) 1 minus the saving rate
XXX        times the income
XXX        over the output
XXX        you know the (.) consumption from
XXX        ((pointing)) step four
XXX        you know output from (.)
XXX        ((pointing)) step two
XXX        and solving this equation you can get
XXX        uh
XXX        this golden rule saving rate
XXX        or you can use
```

XXX the condition that
XXX the investment (.) equals to savings
XXX which is the saving rate (.) times ((writing))
XXX the output
XXX you-you know I star
XXX in step 3
XXX and y star in step 2
XXX so you can use either
XXX these two conditions
XXX and you can find
XXX what is the golden rule saving rate.
XXX so that's the procedure (.) we generally follow
XXX to solve this kind of problem
XXX so now let's look at (.) question 3
XXX ((writing))
XXX so
XXX for part a,
XXX what is the prefect to worker production function
XXX that is the- uh
XXX small y equals to f equals to small k
XXX so
XXX we always derive this
XXX uh
XXX pref- (proficient) worker production function
XXX from the aggregate one
XXX so.
XXX from the setting of this question
XXX y equals to- the capital
XXX y equals to k to point five
XXX ((writing)) LE
XXX ((writing)) the efficient labor
XXX to point five
XXX ((writing)) so by definition,
XXX the small y equals to
XXX the capital one- the capital Y divided by L times E
XXX and from this aggregate production function
XXX it equals to
XXX k to point five
XXX LE (.) to point five
XXX divided by LE.
XXX and you d- do some
XXX eh (.) you know
XXX (signification)
XXX so it becomes
XXX K over LE
XXX to the power of point five
XXX and this K over LE
XXX is just the small k
XXX by definition
XXX is the capital per effective worker.
XXX so it equals to ((writing))
XXX small k to point five

XXX so the pr-production function here
XXX is k to point five.
(12:03)
XXX so that's the result for part a (.3)
XXX ok?
XXX and, (.)
XXX for part b we need to find
XXX the steady state levels of all these four
XXX variables
XXX so we can do it
XXX simply by following the procedures
XXX first (.) we use <u>this</u> condition ((pointing))
XXX to find out what is the value of
XXX capital in steady state
XXX so here
XXX we have a (.) saving rate
XXX and (.) the (formula) this production function
XXX we have just derive it
XXX it is ((writing)) k to point five
XXX I write k star here because
XXX um the capital in the steady state
XXX which is k star satisfy this condition
XXX so
XXX n plus (.) g plus (.) delta the depreciation rate
XXX times
XXX k t- k star
XXX and there is only one unknown in this equation
XXX the k star
XXX so we can solve this (.) equation
XXX so (.) we isolate this k star into the l-
XXX the right hand side
XXX so
XXX it's s over n plus g plus delta
XXX equals to k star to point five
XXX so k star equals to (.) s over
XXX n plus g plus (.) delta
XXX the square of this whole thing.
XXX and we plug the numbers
XXX the values
XXX so these four (.) parameters
XXX into this expression
XXX so it's point two,
XXX over
XXX three percent
XXX three percent and (.) four percent
XXX and square
XXX so the result is
XXX four
XXX ((pause for reading paper))
XXX ((looks back up)) so
XXX this uh
XXX this equation is the only one you need

XXX to solve this (.) k star
XXX ((pause))
XXX so if you- (.)
XXX and in this um equation
XXX means that the savings or the investment
XXX equals to this value
XXX this break even variable ((alt trans: level))
XXX because it's in the steady state
XXX and this will implies that
XXX the delta k equals to zero.
XXX because in steady state,
XXX the capital in this period
XXX and the next- next period
XXX is the same
XXX so the change of the capital
XXX um is zero
XXX so that's where this equation come from.
XXX and if you-
XXX I mean if- if you
XXX can't
XXX understand
XXX where this equation come from now,
XXX I- you have to remember it for the-
(14:59)
XXX just for sake of the coming exam
XXX so
XXX yeah that's four
XXX for the uh capital in steady state and (.2)
XXX for the next step,
XXX in step two we calculate
XXX the output in steady state
XXX so y star equals to
XXX we plug this number into the production function
XXX so (.) k star to the power of point five
XXX so it's four to point five
XXX so it's two.
XXX and in the next step,
XXX we calculate
XXX the value of
XXX I star.
XXX ((writing))
XXX so it equals to n plus g plus (.) delta times k star
XXX we know the value of this four variables
XXX and we can easily gather result is (.2)
XXX point four
XXX and for the consumption,
XXX it equals to output minus investment
XXX so is two minus point four
XXX and the result is 1.6
XXX ((reading paper))
XXX ((looks back up))
XXX so ok

XXX any question?
XXX ((looks around))
XXX and let me do part d first
XXX because we can refer to the procedure
XXX to solving the golden rule levels
XXX and then I will (.) uh
XXX talk about part c
XXX so part d first
XXX in part d we need to find out
XXX all the golden rule levels
XXX of- of all the four variables.
XXX so again,
XXX first
XXX uh actually
XXX five variables
XXX in this case.
XXX so
XXX again,
XXX first we need to (.) find
XXX what is the
XXX um capital stock
XXX in the golden rule.
XXX so.
XXX we use
XXX this condition.
XXX the (margin note for doc) in-
XXX in capital equals to the sum
XXX the summation of these three numbers
XXX so
XXX mpk equals to n plus g plus delta
XXX and with this specific production (.) function form
XXX the mpk equals to point five times
XXX k to minus point five
XXX you just take derivative ((pointing))
XXX with respect to k.
XXX and n is three percent- (.2)
XXX is three percent
XXX three percent for g, and
XXX four percent here
XXX and we can get so it's the
(18:00)
XXX k star pr here
XXX because it's the
XXX uh level in-
XXX in the golden rule
XXX so we can get th-
XXX the result is 25
XXX ((writing))
XXX and
XXX as long as we get
XXX um the steady state
XXX and the- the level for the capital

XXX the other things are straightforward
XXX and in the se-
XXX in the se-
XXX in the second step,
XXX we can compute the- (.) the output.
XXX ((erasing))
XXX so it equals to
XXX ((writing))
XXX capital to the power of point five
XXX so it's five (.) for the output
XXX and for the investment, ((writing))
XXX n plus g plus delta
XXX k star,
XXX so it's (.) two point five
XXX a:nd step four,
XXX is for consumption
XXX is output minus (.) investment
XXX is also two point five
XXX and we have one more step
XXX to calculate
XXX uh
XXX the golden rule saving rate
XXX so let me use this
XXX second one
XXX so from this equation,
XXX it's obvious that
XXX this golden rule saving rate equals to
XXX investment (.) divided by (.) output
XXX and in this example is two point five
XXX over five
XXX so the answer is point five.
XXX ((moves eraser))
XXX and it is exactly the
XXX power of this production function
XXX which is not a coincident
XXX as I have shown this result to you
XXX in last recitation
XXX so (.) yeah that's for part d. (.2)
XXX so this calculation just
XXX regular if you remember all this
XXX ((pointing))
XXX procedures
XXX and the key is
XXX ((pointing))
XXX this two conditions
XXX and if you remember this
XXX ((pointing))
XXX two equations
XXX I mean the following computation is
XXX natural. (.) and straightforward.
XXX so.
XXX now for part c, (.1)

XXX so
XXX let me (.) erase this
XXX ((pause while erasing for a while))
(21:16)
XXX so pa-
XXX in part c,
XXX assume there is initial capital level
XXX which is k one equals to three,
XXX and we need to calculate
XXX that variable in next period
XXX and show it in a graph.
XXX so (.) in steady state
XXX the condition is
XXX delta k equals to zero
XXX but in this part there is no assumption that
XXX um we are in a steady state.
XXX so (.) this- ((crossing out))
XXX this equation doesn't hold
XXX in this case
XXX so
XXX now we need to find out
XXX what is the value for this delta k
XXX because
XXX the capital in next period
XXX noted by k two
XXX equals to k one
XXX the capital in this period
XXX plus delta k.
XXX is
XXX is delta
XXX so in steady state
XXX this number is zero
XXX so this two numbers
XXX these two values coincide with each other
XXX but generally,
XXX without in a steady state,
XXX this delta is not zero.
XXX so
XXX this two numbers are different
XXX and this is the
XXX capital in the next period
XXX and in order to find this value
XXX we need to find out what is delta k because
XXX k one is given
XXX in the question
XXX and ((writing)) as we know
XXX delta k equals to
XXX investment in this period
XXX and that (is noted) as i one
XXX minus the break even level n plus g plus
XXX the depreciation rate
XXX times the capital in this state

```
XXX        k one (.1)
XXX        ((pointing))
XXX        so that is what we know about-
XXX        about delta k
XXX        the change in the capital stock.
XXX        so
XXX        we know k one
XXX        we know n g and delta
XXX        the only unknown is the investment
XXX        and
XXX        remember that the investment
XXX        always equals to the savings
XXX        and the savings
XXX        equals to the saving rate
XXX        times the income
XXX        which is f k.
XXX        and in this case
XXX        is f k one
XXX        because
XXX        the capital, in this state
XXX        is k one.
XXX        and minus
XXX        n plus g plus delta (.2)
XXX        ((erasing))
XXX        times k one
XXX        so here we know the value of
XXX        all this
XXX        variables
XXX        we know s is point two,
XXX        and we know k one we know the form
XXX        of this production function
XXX        we know this three numbers,
XXX        so we just plug (.) them
XXX        into this equation
XXX        so it's point two,
XXX        and it's three,
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(24:00)
```
XXX        k one equals to three,
XXX        to the power of (.) point five,
XXX        minus three percent
XXX        three percent and
XXX        four percent
XXX        times (.) three (.1)
XXX        so the answer is-
XXX        the result is ((writing))
XXX        approx.- approximately
XXX        five percent
XXX        and then we can find the
XXX        capital in the next period
XXX        is- is k one plus ((writing))
XXX        delta k
XXX        three plus
```

XXX five percent
XXX so it's
XXX about three point oh five
XXX so
XXX that is the capital in
XXX steady state
XXX and to show it in a graph,
XXX ((pause for drawing))
XXX we use the
XXX uh
XXX we use the one f- to show the steady state
XXX and first we draw this
XXX break even line,
XXX so it's n plus g plus delta times k
XXX and this ((drawing)) curve for savings
XXX which is
XXX s times the production function
XXX and this (.) ((pointing)) intersection is
XXX the steady state value
XXX so it's k star
XXX and the value is four because
XXX we have got the result from
XXX part d
XXX it's four
XXX it's the same because all the-
XXX the value of all the parameters
XXX s and g and delta is the same
XXX so the steady state is four.
XXX as we calculated in part b
XXX and here we know that the
XXX initial capital stock is
XXX k one equals to three
XXX so. ((pause for drawing))
XXX it's here and
XXX the investment
XXX equals to the savings
XXX from here
XXX the value
XXX at this (.) capital stock is-
XXX this is s times f k one
XXX so it- it is this number point two times
XXX three to one half,
XXX and (.) this number is break even level
XXX is this point.
XXX is the intersection of this vertical line and
XXX when the capital stock is three
XXX and this (.) break even line.
XXX so this value
XXX is ((writing)) n plus g plus delta times k one
XXX and the difference,
XXX between these two values,
XXX is the

XXX delta k
XXX that we calculate
XXX so this is ((drawing)) delta k
XXX and ((pointing)) the capital in next period
XXX equals to
XXX uh
XXX the capital in current period plus delta k
(27:00)
XXX so this is k one, ((writing))
XXX and ((writing))
XXX this is ((writing)) delta k,
XXX so this point is ((writing))
XXX k two
XXX which equals to three point zero five. (.2)
XXX and
XXX in steady state,
XXX the difference of this two values is zero.
XXX right?
XXX they coin- c- coincide
XXX with each other
XXX so that's why we use the condition
XXX delta k equals to zero
XXX for the steady state.
XXX and now, for
XXX for any given value
XXX this may not be the case
XXX so we should
XXX uh calculate in this way
XXX first find out
XXX what is this uh investment or a savings
XXX and then we find out what is the delta k
XXX and then we plug-
XXX we add this delta k into the
XXX current capital stock
XXX and we can find the capital stock
XXX in the next period.
XXX ((pause))
XXX ok
XXX any questions (class,)
XXX ((looking at board and paper))
XXX so
XXX now
XXX the last part
XXX ((erasing for about 10 seconds))
XXX ((writing))
XXX °so ((pause))
XXX part e,
XXX now suppose
XXX the saving rate
XXX um
XXX changes from
XXX s equal to point two,

XXX		to the golden rule saving rate.
XXX		so before this change,
XXX		the saving rate is
XXX		zero point two,
XXX		which is the value that we use in part b
XXX		to calculate all of this variables
XXX		and
XXX		now it turns to ((writing)
XXX		the golden rule saving rate
XXX		which is ((writing))
XXX		zero point five,
XXX		as we have calculated in
XXX		part d. ((looking at paper))
XXX		and we need to find out
XXX		the immediate effect on
XXX		income per effective worker
XXX		a:nd consumption.
XXX		and
XXX		also the long- the long run effect
XXX		of these two variables
XXX		so first let me
XXX		look at the
XXX		long run effect
XXX		((writing))
XXX		as we know in the long run,
XXX		uh: the economy will always converges into the
XXX		steady state
(30:00)
XXX		so ((pointing))
XXX		in order-
XXX		in order to find out what is the long run effect
XXX		we only need to compare
XXX		this two numbers
XXX		uh in the steady state
XXX		with this old uh saving rate
XXX		we have a steady state value for
XXX		uh
XXX		for the- uh for the output
XXX		or the i- or the income, a:nd
XXX		the consumption
XXX		so.
XXX		when s equals to point two,
XXX		((writing)) y star equals to
XXX		((glancing at the board))
XXX		it's two
XXX		and consumption is one point six
XXX		((writing))
XXX		and when ((writing))
XXX		delta saving rate is-
XXX		((erasing))
XXX		is the
XXX		golden rule one

XXX which is
XXX zero point five,
XXX ((writing))
XXX uh the
XXX the g-
XXX the output is
XXX ((glancing at board))
XXX five?
XXX °if I remember,
XXX ((looks back at where he's writing))
XXX it's five
XXX and the consumption is
XXX two point five.
XXX because in the long run, ((pointing))
XXX uh the economy will always converge to the
XXX steady state
XXX so
XXX it is efficient for us to compare
XXX ((pointing))
XXX this- this two numbers
XXX and these two numbers
XXX to figure out the long run effect
XXX so
XXX so it is (.) obvious that
XXX when-
XXX when the saving rate increase to the
XXX uh golden rule level
XXX the
XXX output will increase
XXX and the consumption will also increase
XXX so that is the
XXX long run effect
XXX both the
XXX output per effective worker and the
XXX consumption (.) per effective worker will
XXX increase
XXX ((writing))
XXX so it's-
XXX so remember that
XXX for the long run effect
XXX you should always compare the (.2)
XXX values in the steady state
XXX between this two different saving rate.
XXX ((writing))
XXX ((looking at paper))
XXX and then let me
XXX show us the effect
XXX immediately
XXX so the immediate effect
XXX ((pause for writing))
XXX so
XXX typically when

```
XXX        the saving rate is different
XXX        and ((pointing))
XXX        in the- in the (current) state
XXX        when the- ((pointing))
XXX        when there is a change in the saving rate
XXX        the sto- uh
XXX        I mean the s- the
XXX        capital stock
XXX        will be the same.
XXX        a:nd your-
XXX        the household will-
XXX        will change its consumption and investment
XXX        and in the next period
XXX        uh
XXX        the different investment will work on this capital
XXX        so the capital will begin to change
XXX        so
XXX        ((writing))
(33:00)
XXX        I mean to illustrate in this graph,
XXX        ((drawing))
XXX        so this is the break even line
XXX        and this is the (.1)
XXX        savings with the old saving rate
XXX        which is
XXX        let me de-denote it by uh
XXX        as one
XXX        and now if there is an increase in this
XXX        saving rate
XXX        right?
XXX        so
XXX        this curve will (.) level up,
(33:24)
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