Chapter 10. Making Sense of Quantum Mechanics

This (very cryptic) chapter addresses the issues of quantum mechanics interpretation that are still a subject of debate – fortunately not affecting practical applications of the quantum theory.

10.1. Hidden variables and local reality

Only now, with a quantitative understanding of the principles of quantum mechanics, we are ready to proceed to the discussion of its interpretation – the issue which is very closely related to problems of measurements, already discussed in Sec. 7.7. As was already mentioned in that section, the founding fathers of quantum mechanics have not left much guidance on these topics, because in the first years after the advent of this exciting new theory they gave understandable preference to using it for deriving new particular results, and then were much distracted by the development of nuclear physics and its applications. This is why, after a very important but inconclusive discussion between A. Einstein and N. Bohr in the mid-30s, the debates of quantum measurements and the related conceptual issues of quantum mechanics have resumed only in the 1950s. They have led to a key contribution by J. Bell in the early 1960s, and an important experimental work on verifying Bell’s inequalities (see below), but besides that work, the recent progress is marginal, and opinions of even prominent physicists on certain issues are still very much different.

Perhaps the central controversial issue is question (iii) posed in Sec. 7.7: what (if any :-) is the “real” state of a quantum-mechanical system before a nearly-perfect measurement giving a certain outcome? In order to be specific, let us focus again on the simplest example of Stern-Gerlach measurements of spin-½ particles - because of their physical transparency and technical simplicity. As the reader knows very well by now, even in a pure quantum spin state (for example, $\uparrow$), i.e. the least uncertain state of the system, the results of the Stern-Gerlach measurements of other spin component are still uncertain. Indeed, as we know from Sec. 4.4, the ket-vector of this state may be presented as

$$|\uparrow\rangle = \frac{1}{\sqrt{2}}(|\rightarrow\rangle + |\leftarrow\rangle),$$

so that the probabilities of measuring any of values $S_x^+ = +\hbar/2$ and $S_x^- = -\hbar/2$ equal 50%. So, has the spin had a certain value of $S_x$ a split second before the Stern-Gerlach measurement that gave a certain outcome, for example $S_x^+ = +\hbar/2$? For a classical system, with perfect detectors, the answer is definitely yes. In this case, the pre-measurement probability of 50% just reflects the degree of our ignorance about the real state of the system, and the detector merely reveals it.

However, the situation in quantum mechanics is different, and such interpretation is impossible, as was clearly shown in the famous EPR paper published in 1935 by A. Einstein, B. Podolsky, and N.

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1 I believe that another popular name for this group of issues, “foundations of quantum mechanics”, is hardly appropriate. The only reliable foundation of physics (or any other genuine scientific discipline) is a set of experimental facts.

2 As was discussed in Sec. 7.7, Stern-Gerlach-type experiments may be readily made almost “perfect”, i.e. virtually unaffected by instrument imperfections, provided that we do not care about the state of the particle after a single-shot measurement.
Rosen. Its original discussed thought experiments with a pair of 1D particles prepared in a quantum state in that both the sum of their momenta and difference of their coordinates are exactly fixed: \( p_1 + p_2 = 0, x_1 - x_2 = a \). However, usually the discussion is recast into an equivalent Stern-Gerlach experiment shown in Fig. 1a.\(^4\) A source emits rare pairs of spin-\( \frac{1}{2} \) particles, propagating in opposite directions, with exactly zero net spin, but otherwise in random spin states. After the spatial separation of the particles has become sufficiently large (see below), the spin state of each of them is measured with a Stern-Gerlach detector, one of them (Fig. 1, detector SG\(_1\)) somewhat closer to the particle source, so it makes the measurement first, at time \( t_1 < t_2 \).

![Diagram of Stern-Gerlach experiment](image)

First, let the detectors be oriented say along the same direction, say axis \( z \). Evidently, the probability of each detector to give any of values \( S_z = \pm \hbar/2 \) is 50\%. However, if the first detector had given result \( S_z = -\hbar/2 \), even before the second detector’s measurement, we know that it will give result \( S_z = +\hbar/2 \) with 100\% probability. So far, the result allows for a classical interpretation, just for the single-particle measurements discussed in Secs. 2.5 and 7.7. Thus we may fancy that the second particle really has a definite spin before the measurement, and the first measurement has just removes our ignorance about that reality. In other words, the change of probability is due to the statistical ensemble redefinition: the 50\% probability belongs to the ensemble of all experiments, while the 100\% probability, to the sub-ensemble of experiments with the \( S_z = -\hbar/2 \) outcome of the first experiment.

However, let the source generate the particle pairs in the entangled, singlet state (8.19),

\[
|s_{12}\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right),
\]

that certainly satisfies the above assumptions: the probability of each \( S_z \) value of any particle is 50\%, the sum of both \( S_z \) is exactly zero, and if the first detector’s result is \( S_z = -\hbar/2 \), then the state of the remaining particle is \( \uparrow \), with zero uncertainty. Now let us use Eq. (1), and its counterpart for vector \( |\downarrow\rangle \),\(^5\) to present the same initial state (2) in the form.

\(^3\) This is possible, because the corresponding operators commute: \([\hat{p}_1 - \hat{p}_2, \hat{x}_1 + \hat{x}_2] = [\hat{p}_1, \hat{x}_1] - [\hat{p}_2, \hat{x}_2] = 0 \).

\(^4\) Another convenient experimental technique of entangled state generation, frequently used in this field, is the four-wave mixing (FWM) of optical photons. Its brief discussion may be found, for example, in CM Sec. 5.5.

\(^5\) As a reminder, it differs from Eq. (1) only by the sign in the parentheses - see, e.g., Eqs. (4.123).
\[ |s_{12} \rangle = \frac{1}{\sqrt{2}} \left[ \frac{1}{\sqrt{2}} (|\rightarrow\rangle + |\leftarrow\rangle) \frac{1}{\sqrt{2}} (|\rightarrow\rangle - |\leftarrow\rangle) - \frac{1}{\sqrt{2}} (|\rightarrow\rangle - |\leftarrow\rangle) \frac{1}{\sqrt{2}} (|\rightarrow\rangle + |\leftarrow\rangle) \right]. \] (10.3)

Opening the parentheses (without swapping the ket-vector order!), we get an expression similar to Eq. (2), but now for the \( x \)-basis:

\[ |s_{12} \rangle = \frac{1}{\sqrt{2}} (|\rightarrow\leftarrow\rangle - |\leftarrow\rightarrow\rangle). \] (10.4)

Hence if we use the first detector (closest to the particle source) to measure \( S_x \) rather than \( S_z \), then after it had given as certain result (say, \( S_x = -\hbar/2 \)), we know for sure, before the second particle spin’s measurement, that its \( S_x \) component equals \( +\hbar/2 \).

So, depending on the experiment performed on the first particle, the second particle turns out to be in one of two states - either with a definite component \( S_z \) or with a definite component \( S_x \), in each case without any uncertainty. Evidently, this situation cannot be interpreted in classical terms if the particles do not interact during the measurements. A. Einstein in was deeply unhappy with such situation, because it did not satisfy the general requirement to any theory, which nowadays is called the local reality. His definition of this requirement was as follows:

“The real factual situation of system 2 is independent of what is done with system 1 that is spatially separated from the former”.

(Here the term “separated” in this sentence is a bit uncertain, but from the context it is clear that Einstein meant the detector separation by a superluminal interval, i.e. by distance

\[ |r_1 - r_2| > c|t_1 - t_2|, \] (10.5)

where the measurement time difference, participating in the right-hand part, includes the measurement duration.) In Einstein’s view, since quantum mechanics does not satisfy the local reality condition, it cannot be considered a complete theory of Nature.

This situation naturally raises the question whether something (usually called hidden variables) may be added to the quantum-mechanical description in order to satisfy the local reality requirement. The first definite statement in this regards was J. von Neumann’s “proof”\(^6\) (first famous, then infamous :-) that such variables cannot be introduced; for a while his work satisfied quantum mechanics practitioners.\(^7\) A major new contribution to the problem was made only in the 1960s by J. Bell.\(^8\) First of all, he has found an elementary (in his words, “foolish”) error in von Neumann’s logic, which voids his “proof”. Second, he demonstrated that Einstein’s local reality condition is incompatible with conclusions of quantum mechanics – that had been, by that time, confirmed by too many experiments to be seriously questioned. Since no hidden variable introduction can change this situation, in this sense such introduction is impossible.

\(^6\) In his pioneering book J. von Neumann, *Mathematische Grundlagen der Quantenmechanik* [Mathematical Foundations of Quantum Mechanics], Springer, 1932. (The first English translation was published only in 1955.)

\(^7\) Evidently, it would not satisfy A. Einstein, but reportedly he did not know about von Neumann’s result before signing the EPR paper.

Let me describe a particular version of Bell’s proof (suggested by E. Wigner), using the same EPR pair experiment (Fig. 1a), in that each SG detector may be oriented in any of 3 directions: \(a\), \(b\), or \(c\) - see Fig. 1b. As we know from Chapter 4, if a fully-polarized beam of spin-\(\frac{1}{2}\) particles is passed through a Stern-Gerlach apparatus forming angle \(\phi\) with the polarization axis, the probabilities of two counterpart outcomes of the experiment are

\[
W(\phi_+) = \cos^2 \frac{\phi}{2}, \quad W(\phi_-) = \sin^2 \frac{\phi}{2}. \tag{10.6}
\]

Let us use this formula to calculate all joint probabilities of measurement outcomes, starting from the detectors 1 and 2 oriented, respectively, in directions \(a\) and \(c\). Since the angle between negative direction of axis \(a\) and positive direction of axis \(c\) is \(\phi_{a^+,c^+} = \pi - \phi\) (see the dashed arrow in Fig. 1b), we get

\[
W(a_+,c_+) = W(a_-)W(c_+|a_+) = \frac{1}{2} \cos^2 \frac{\phi_{a^-,c^+}}{2} = \frac{1}{2} \cos^2 \frac{\pi - \phi}{2} = \frac{1}{2} \sin^2 \frac{\phi}{2}. \tag{10.7}
\]

Absolutely similarly,

\[
W(c_+,b_+) = W(c_-)W(b_+|c_+) = \frac{1}{2} \sin^2 \frac{\phi}{2}, \tag{10.8}
\]

\[
W(a_+,b_+) = W(a_-)W(b_+|a_+) = \frac{1}{2} \cos^2 \frac{\pi - 2\phi}{2} = \frac{1}{2} \sin^2 \phi. \tag{10.9}
\]

Now note that for any angle \(\phi\) smaller than \(\pi/2\) (as in the case shown in Fig. 1b),

\[
\frac{1}{2} \sin^2 \phi \geq \frac{1}{2} \sin^2 \frac{\phi}{2} + \frac{1}{2} \sin^2 \frac{\phi}{2} = \sin^2 \frac{\phi}{2}. \tag{10.10}
\]

(For example, for \(\phi \to 0\) the left-hand part of this relation tends to \(\phi^2/2\), while the right-hand part, to \(\phi^2/4\).) Hence the quantum-mechanical result gives, in particular,

\[
W(a_+,b_+) \geq W(a_+,c_+) + W(c_+,b_+), \quad \text{for } |\phi| \leq \pi/2. \tag{10.11}
\]

On the other hand, we may compose another inequality for the same probabilities without calculating them from any particular theory, but using the local reality assumption. Let us list all possible outcomes of detector measurements, taking into account the zero net spin:

<table>
<thead>
<tr>
<th>Detector 1 results</th>
<th>Detector 2 results</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_+, b_+, c_+)</td>
<td>(a_+, b_+, c_+)</td>
<td>(W_1)</td>
</tr>
<tr>
<td>(a_+, b_+, c_+)</td>
<td>(a_+, b_+, c_+)</td>
<td>(W_2)</td>
</tr>
<tr>
<td>(a_+, b_+, c_+)</td>
<td>(a_+, b_+, c_+)</td>
<td>(W_3)</td>
</tr>
<tr>
<td>(a_+, b_+, c_+)</td>
<td>(a_+, b_+, c_+)</td>
<td>(W_4)</td>
</tr>
<tr>
<td>(a_+, b_+, c_+)</td>
<td>(a_+, b_+, c_+)</td>
<td>(W_5)</td>
</tr>
<tr>
<td>(a_+, b_+, c_+)</td>
<td>(a_+, b_+, c_+)</td>
<td>(W_6)</td>
</tr>
<tr>
<td>(a_+, b_+, c_+)</td>
<td>(a_+, b_+, c_+)</td>
<td>(W_7)</td>
</tr>
<tr>
<td>(a_+, b_+, c_+)</td>
<td>(a_+, b_+, c_+)</td>
<td>(W_8)</td>
</tr>
</tbody>
</table>
From the local reality point of view, these measurement options are independent, so we may write:

\[ W(a_+, c_+) = W_2 + W_4, \quad W(c_+, b_+) = W_3 + W_7, \quad W(a_+, b_+) = W_3 + W_4. \tag{10.12} \]

On the other hand, since no probability may be negative (by its very definition), we may always write

\[ W_3 + W_4 \leq (W_2 + W_4) + (W_3 + W_7). \tag{10.13} \]

Plugging into this inequality the values of these two parentheses, given by Eq. (12), we get

\[ W(a_+, b_+) \leq W(a_+, c_+) + W(c_+, b_+). \tag{10.14} \]

This is (one of several possible forms of) the Bell’s inequality that has to be satisfied by any local-reality theory; it directly contradicts the quantum-mechanical result (11).

Though experimental tests of the Bell’s inequalities had been started in the late 1960s, the interpretation of first results was vulnerable to two criticisms:

(i) The detectors were not fast enough and not far enough to have relation (5) satisfied. This is why, as the matter of principle, there was a chance that information on one measurement had been transferred (by some, mostly implausible) means to particles before the second measurement - the so-called locality loophole.

(ii) Particle detection efficiencies were too low to have sufficiently small error bars for both parts of the inequality – the detection loophole.

Gradually, these loopholes have been closed. As expected, substantial violations of Bell inequalities equivalent to Eq. (14) have been proved, essentially rejecting any possibility to reconcile quantum mechanics with Einstein’s local reality requirement.

### 10.2. Interpretations of quantum mechanics

The fact that quantum mechanics is incompatible with local reality, makes it reconciliation with our (classically-bred) “common sense” rather challenging. Here is a brief list of the major interpretations of quantum mechanics, that try to provide at least a partial reconciliation of this kind:

(i) The so-called Copenhagen interpretation, to which most physicists subscribe. This “interpretation” does not really interpret anything; it just states the internal randomness of measurement results in quantum mechanics, essentially saying: “Do not worry; this is just how it is; live with it”. For me personally, this interpretation, at least in its most frequently repeated forms, has only one, rather pedagogical weakness: though it implies statistical ensembles (otherwise, how would you define the probability?), but does not put a sufficient emphasis on their role, in particular the possible ensemble

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9 Important milestones on that way were experiments by A. Aspect et al., Phys. Rev. Lett. 49, 91 (1982) and M. Rowe et al., Nature 409, 791 (2001). A detailed review of the experimental situation was given, for example, by M. Genovese, Phys. Repts. 413, 319 (2005); see also more recent experiments by D. Matsukevich et al., Phys. Rev. Lett. 100, 150404 (2008) and D. Salart et al., Nature 454, 861 (2008). Presently, a low-noise demonstration of the Bell inequality violation has become a standard test in each experiment with entangled qubits used for quantum encryption research – see Sec. 8.5.
redefinition as the only key point of human involvement in the measurement process.\textsuperscript{10} Perhaps the most impressive objection to the Copenhagen interpretation was given by A. Einstein during his 1935 discussion with N. Bohr: “God does not play dice.” OK, when Einstein speaks, we all should listen, but perhaps when God speaks (through the experimental results), we have to pay even more attention.

(ii) \textit{Non-local reality}. After the dismissal of von Neumann’s “proof” by J. Bell, to the best of my knowledge, there has been no proof that hidden parameters could not be introduced, provided that they do not imply the local reality. Of constructive approaches, perhaps the most notable contribution was made by D. Bohm\textsuperscript{11} who developed the L. de Broglie’s interpretation of the wavefunction as a “pilot wave”, making it quantitative. In the wave mechanics version of this concept, the wavefunction, governed by the Schrödinger equation, just guides a real, point-like classical particle whose coordinates serve as hidden variables. However, this concept does not satisfy the notion of local reality. Namely, the measurement of particle’s coordinate at a certain point \( r_1 \) has to \textit{instantly} change the wavefunction everywhere, including points \( r_2 \) in the superluminal interval range (4). So, Bohm’s hidden variables would hardly make A. Einstein happy. After having recognized this problem, D. Bohm abandoned his theory – in J. Bell’s view, perhaps too early. In my personal taste, however, the assumption of such (in Einstein’s words) “spooky action at a distance” is too large a sacrifice to save the classical determinism.

(iii) The \textit{many-world interpretation} introduced in 1957 by H. Everitt and popularized in the 1960s and 1970s by B. de Witt. In this interpretation, \textit{all} possible measurement outcomes do happen, splitting the Universe into the corresponding number of “parallel” Universes, so that from one of them, other Universes and hence other outcomes cannot be observed. Let me leave to the reader an estimate of the rate at which the parallel Universes being constantly generated (say, per second), taking into account that such generation should take place not only at explicit lab experiments, but at any irreversible process such as fission of any atom nucleus or absorption of a photon, everywhere in each Universe – whether its result is recorded or not. Even the main proponent of this interpretation, B. de Witt, has confessed: “The idea is not easy to reconcile with common sense”. I agree.

(iv) \textit{The quantum logic}. In desperation, some physicists turned philosophers have decided to dismiss the very logic we are using – in science and elsewhere, so that a statement like “the Bell inequalities are violated” would not make any definite sense. OK, if we dismiss the formal logic, I do not know how we can use any scientific theory and make any predictions - until the quantum logic experts tell us what to replace the classical logic with. To the best of my knowledge, so far they have not done that, at least for the measurement process. I personally trust J. Bell’s opinion: “It is my impression that the whole vast subject of Quantum Logic has arisen […] from the misuse of a word.”

The weakness of all interpretations of quantum mechanics is that, as far as I know, neither of them has yet provided any suggestion how this particular interpretation might be tested experimentally to exclude other ones. On the positive side, there is a consensus that quantum mechanics makes correct, if sometimes probabilistic, predictions of all reliable experimental results we are aware of. Maybe, this is not that bad for a scientific theory.\textsuperscript{12}

\textsuperscript{10} A detailed discussion of statistical ensemble’s role may be found, e.g., in L. Balentine, Quantum Mechanics, World Scientific, 1998.


\textsuperscript{12} If the reader is not satisfied with this “positivistic” approach, and wants to improve the situation, my earnest advice would be to start not from square one, but from reading what other (including some very clever!) people thought about it. A good starting point is the review collection by J. Wheeler and W. Zurek (eds.), \textit{Quantum Theory and Measurement}, Princeton U. Press, 1983.