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Front Matter

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Essential Graduate Physics

Lecture Notes and Problems

Beta version

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Beta version, December 2013
with later problem additions and error corrections

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Preface

This is a series of lecture notes and problems on "Essential Graduate Physics", consisting of the following four parts:

CM: *Classical Mechanics* (for a 1-semester course),
EM: *Classical Electrodynamics* (2 semesters),
QM: *Quantum Mechanics* (2 semesters), and
SM: *Statistical Mechanics* (1 semester).

The parts share a teaching style, structure, and (with a few exceptions) notation, and are interlinked by extensive cross-referencing. I believe that due to this unity, the notes may be used for teaching these courses not only in the (preferred) sequence shown above but in almost any order – or in parallel.

Each part is a two-component package consisting of:

(i) *Lecture Notes* chapter texts,[2] with a list of exercise problems at the end of each chapter, and
(ii) *Exercise and Test Problems with Model Solutions* files.

The series also includes this front matter, two brief reference appendices, *MA: Selected Mathematical Formulas* (16 pp.) and *UCA: Selected Units and Constants* (4 pp.), and a list of references (2 pp.)

The series is a by-product of the so-called *core physics* courses I taught at Stony Brook University from 1991 to 2013. Reportedly, most physics departments require their graduate students to either take a set of similar courses or pass comprehensive exams based on an approximately similar body of knowledge. This is why I hope that my notes may be useful for both instructors and students of such courses, as well as for individual learners.

The motivation for composing the lecture notes (which had to be typeset because of my horrible handwriting) and their distribution to Stony Brook students was my desperation to find textbooks I could actually use for teaching. First of all, the textbooks I could find, including the most influential *Theoretical Physics* series by Landau and Lifshitz, did not match my class audiences, which included experiment-oriented students, some PhD candidates from other departments, some college graduates with substandard undergraduate backgrounds, and a few advanced undergraduates. Second, for the rigid time restrictions imposed on the core physics courses, most available textbooks are way too long, and using them would mean hopping from one topic to another, picking up a chapter here and a section there, at a high risk of losing the necessary background material and logical connections between course components – and students' interest with them. On the other hand, many textbooks lack even brief discussions of several traditional and modern topics that I believe are necessary parts of *every* professional physicist's education.[3,4]

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[2] The texts are saved as separate .pdf files of each chapter, optimized for two-page viewing and double-side printing; merged files for each part and the series as a whole, convenient for search purposes, are also provided.

[3] To list just a few: statics and dynamics of elastic and fluid continua, basic notions of physical kinetics, turbulence and deterministic chaos, physics of reversible and quantum computation, energy relaxation and dephasing of open quantum systems, the van der Pol method (a.k.a. the Rotating-Wave Approximation, RWA) in classical and quantum mechanics, physics of electrons and holes in semiconductors, the weak-potential and tight-binding approximations in the energy band theory, optical fiber electrodynamics, macroscopic quantum effects in

The main goal of my courses was to make students familiar with the basic notions and ideas of physics (hence the series' title), and my main effort was to organize the material in a logical sequence the students could readily follow and enjoy, at each new step understanding why exactly they need to swallow the next knowledge pill. As a backside of such a minimalistic goal, I believe that my texts may be used by advanced undergraduate physics students as well. Moreover, I hope that selected parts of the series may be useful for graduate students of other disciplines, including astronomy, chemistry, mechanical engineering, electrical, computer and electronic engineering, and material science.

At least since Confucius and Sophocles, i.e. for the past 2,500 years, teachers have known that students can master a new concept or method only if they have seen its application to at least a few particular situations. This is why in my notes, the range of theoretical physics methods is limited to the approaches that are indeed necessary for the solution of the problems I had time to discuss, and the introduction of every new technique is always accompanied by an application example or two. Additional exercise problems are listed at the end of each chapter of the lecture notes; they may be used for homework assignments. Individual readers are strongly encouraged to solve as many of these problems as possible.[5]

Detailed model solutions of the exercise problems (some with additional expansion of the lecture material), and several shorter problems suitable for tests (also with model solutions), are gathered in six separate files – one per semester. These files are available for both university instructors and individual readers – free of charge, but in return for a signed commitment to avoid unlimited distribution of the solutions – see p. vii below. For instructors, these files are available not only in the Adobe Systems' Portable Document Format (*.pdf) but also in the Microsoft Office 1997-2003 format (*.doc) free of macros, so that the problem assignments and solutions may be readily grouped, edited, etc., before their distribution to students, using either virtually any version of Microsoft Word or independent software tools – e.g., the public-domain OpenOffice.org.

I know that my texts are far from perfect. In particular, some sacrifices made in the topic selection, always very subjective, were extremely painful. (Most regretfully, I could not find time for even a brief introduction to general relativity.[6]) Moreover, it is almost certain that despite all my efforts and the great help from SBU students and teaching assistants, not all typos/errors have been weeded out. This is why all remarks (however candid) and suggestions by the readers would be highly appreciated; they may be sent to *klikharev@gmail.com*. All significant contributions will be gratefully acknowledged in future editions of the series.

Bose-Einstein condensates, Bloch oscillations and Landau-Zener tunneling, cavity QED, and the Density Functional Theory (DFT). All these topics are discussed, if only concisely, in these notes.

[4] Recently, several high-quality graduate-level teaching materials became available online, including M. Fowler's *Graduate Quantum Mechanics Lectures* (http://galileo.phys.virginia.edu/classes/751.mf1i.fall02/home.html), R. Fitzpatrick's text on *Classical Electromagnetism* (farside.ph.utexas.edu/teaching/jk1/Electromagnetism.pdf), B. Simons' lecture notes on *Advanced Quantum Mechanics* (www.tcm.phy.cam.ac.uk/~bds10/aqp.html), and D. Tong's lecture notes on several topics (www.damtp.cam.ac.uk/user/tong/teaching.html).

[5] The problems that require either longer calculations or more creative approaches (or both) are marked by asterisks.

[6] For an introduction to that subject, I can recommend either its review by S. Carroll, *Spacetime and Geometry*, Addison-Wesley, 2003, or a longer text by A. Zee, *Einstein Gravity in a Nutshell*, Princeton U. Press, 2013.

Disclaimer

Since these materials are available free of charge, it is hard to imagine somebody blaming their author for deceiving "customers" for his commercial gain. Still, I would like to go a little bit beyond the usual litigation-avoiding claims,[7] and offer a word of caution to potential readers, to preempt their possible later disappointment.

This is NOT a course of theoretical physics – at least in the contemporary sense of the term

Though much of the included material is similar to that in textbooks on "theoretical physics" (most notably in the famous series by L. Landau and E. Lifshitz), this lecture note series is different from them by its focus on the basic concepts and ideas of physics, their relation to experimental data, and most important applications – rather than on sophisticated theoretical techniques. Indeed, the set of theoretical methods discussed in the notes is limited to the minimum necessary for quantitative understanding of the key notions of physics and for solving a few (or rather about a thousand :-) core problems. Moreover, because of the notes' shortness, I have not been able to cover some key fields of theoretical physics, most notably general relativity and quantum field theory – beyond some introductory elements of quantum electrodynamics in QM Chapter 9. If you want to work in modern theoretical physics, you need to know much more than what this series teaches!

Moreover, this is NOT a textbook – at least not the usual one

A usual textbook tries (though most commonly fails) to cover virtually all aspects of the addressed field. As a result, it is typically way too long to be fully read and understood by students during the time allocated for the corresponding course, so the instructors are forced to pick up selected chapters and sections, frequently losing the narrative's logic lines. In contrast, these notes are much shorter (about 200 pages per semester), enabling their thorough reading – perhaps with just a few later sections dropped, depending on the reader's interests. I have tried to mitigate the losses due to this minimalistic approach by providing quite a few solved problems and extensive further reading recommendations on the topics I had no time to cover. The reader is highly encouraged to use these sources (and/or the corresponding chapters of more detailed textbooks) on any topics of their special interest.

Then, what do these notes ARE and why you may like to use them – I think

By tradition, graduate physics education consists of two main components: research experience and advanced physics courses. Unfortunately, the latter component is currently under pressure in many physics departments, apparently because of two reasons. On one hand, the average knowledge level of the students entering graduate school is not improving, so bringing them up to the level of contemporary research becomes increasingly difficult. On the other hand, the research itself is becoming more

[7] Yes, Virginia, these notes represent only my personal opinions, not necessarily those of the Department of Physics and Astronomy of Stony Brook University, the SBU at large, the SUNY system as a whole, the Empire State of New York, the federal agencies and private companies that funded my group's research, etc. No dear, I cannot be held responsible for any harm, either bodily or mental, their reading may (?) cause.

fragmented, so the students frequently do not feel an immediate need for a broad physics knowledge base for their PhD project success. Some thesis advisors, trying to maximize the time they could use their students as a cheap laboratory workforce, do not help.

I believe that this trend toward the reduction of broad physics education in graduate school is irresponsible. Experience shows that during their future research career, a typical current student will change their research fields several times. Starting from scratch in a new field is hard – terribly hard at an advanced age (believe me :-). However, physics is fortunate to have a stable *hard core* of knowledge, which many other sciences lack. With this knowledge, students will always feel in physics at home, while without it, they may not be able even to understand research literature in the new field, and would risk being reduced to auxiliary work roles – if any at all.

I have seen the main objective of my Stony Brook courses to give an introduction to this core of physics, at the same time trying to convey my own enchantment by the unparalleled beauty of the concepts and ideas of this science, and the remarkable logic of their fusion into a wonderful single construct. Let me hope that these notes relay not only the knowledge as such but also at least a part of this enchantment.

Acknowledgments

I am extremely grateful to my faculty colleagues and other readers who commented on certain sections of the notes; here is their list (in the alphabetic order):[8]

A. Abanov, P. Allen, D. Averin, S. Berkovich, P.-T. de Boer, M. Fernandez-Serra,
R. F. Hernandez, P. Johnson, T. Konstantinova, A. Korotkov, V. Semenov, F. Sheldon,
S. Sridharan, E. Tikhonov, O. Tikhonova, X. Wang, T.-C. Wei.

(Obviously, they are not responsible for the remaining deficiencies.)

The Department of Physics and Astronomy of Stony Brook University was very responsive to my kind requests for certain time ordering of my teaching assignments, which was beneficial for note writing and editing. The department, and the university as a whole, also provided a very friendly general environment for my work there for almost three decades.

A large part of my scientific background and research experience reflected in these materials came from my education (and then work) in the Department of Physics of Moscow State University.

And last but not least, I would like to thank my wife Lioudmila for several good bits of advice on aesthetic aspects of note typesetting, and more importantly for all her love, care, and patience – without them, this writing project would be impossible.

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[8] I am very sorry that I have not kept proper records from the beginning of my lectures at Stony Brook, so I cannot list all the numerous students and TAs who had kindly attracted my attention to typos in earlier versions of these notes. Needless to say, I am very grateful to them all as well.

Problem Solution Request Templates

Requests should be sent to either *klikharev@gmail.com* or *konstantin.likharev@stonybrook.edu* in either of the following forms:

- an email from a valid university address,
- a scanned copy of a signed letter – as an email attachment.

Approximate contents:

A. Request from a Prospective Instructor

Dear Dr. Likharev,

I plan to use your lecture notes and problems of the *Essential Graduate Physics* series, part(s) <select: CM, EM, QM, SM>, in my course <title> during <semester, year> in the <department, university>. I would appreciate your sending me the file(s) *Exercise and Test Problems with Model Solutions* of that part(s) of the series in the <select: .pdf, both .doc and .pdf> format(s).

I will avoid unlimited distribution of the solutions, in particular their posting on externally searchable websites. If I distribute the solutions among my students, I will ask them to adhere to the same restraint.

I will let you know of any significant typos/deficiencies I may find.

Sincerely, <signature, full name, university position, work phone number>

B. Request from an Individual Learner

Dear Dr. Likharev,

I plan to use your lecture notes and problems of the *Essential Graduate Physics* series, part(s) <select: CM, EM, QM, SM>, for my personal education. I would appreciate your sending me the file(s) *Exercise and Test Problems with Model Solutions* of that part(s) of the series.

I will not share the material with anyone, and will not use it for passing courses that are officially based on your series.

I will let you know of any significant typos/deficiencies I may find.

Sincerely, <signature, full name, present home address (in English), acting phone number>

Notation

Abbreviations

Eq. any formula (e.g., equality)

Fig. figure

Sec. section

c.c. complex conjugate

h. c. Hermitian conjugate

Fonts

F, $\mathcal{F}$ scalar variables[9]

$\mathbf{F}$, $\boldsymbol{\mathcal{F}}$ vector variables

$\hat{F}$, $\hat{\mathcal{F}}$ scalar operators

$\hat{\mathbf{F}}$, $\hat{\boldsymbol{\mathcal{F}}}$ vector operators

F matrix

$F_{jj'}$ matrix element

Symbols

$\dot{}$ time differentiation operator (d/dt)

∇ spatial differentiation vector (*del*)

$\approx$ approximately equal to

$\sim$ of the same order as

$\propto$ proportional to

$\equiv$ equal to by definition (or evidently)

$\cdot$ scalar ("dot-") product

$\times$ vector ("cross-") product[10]

$\bar{}$ time averaging

$\langle\ \rangle$ statistical averaging

[,] commutator

{ , } anticommutator

$\mathbf{n}$ unit vector

Parts of the series

CM: Classical Mechanics

EM: Classical Electrodynamics

QM: Quantum Mechanics

SM: Statistical Mechanics

Appendices

MA: Selected Mathematical Formulas

UCA: Selected Units and Constants

Prime signs

The prime signs (′, ″, etc.) are used to distinguish similar variables or indices (such as j and j' in the matrix element above), rather than to denote derivatives.

Italics

In the text, Italic fonts are used for emphasis – in particular, of the new terms at their first usage.

Frames

The most general and/or important formulas are highlighted with blue frames and short titles on the margins.

Numbering

Chapter numbers are dropped in all references to formulas, figures, footnotes, and problems within the same chapter.

[9] The same letter, typeset in different fonts, typically denotes different variables.

[10] On a few occasions, the cross sign is used to emphasize the usual multiplication of scalars.

General Table of Contents

CM: Classical Mechanics	Pages	Exercise Problems
Table of Contents	4	–
Chapter 1. Review of Fundamentals	14	14
Chapter 2. Lagrangian Analytical Mechanics	14	11
Chapter 3. A Few Simple Problems	22	27
Chapter 4. Rigid Body Motion	32	37
Chapter 5. Oscillations	38	22
Chapter 6. From Oscillations to Waves	30	26
Chapter 7. Deformations and Elasticity	38	23
Chapter 8. Fluid Mechanics	30	27
Chapter 9. Deterministic Chaos	14	5
Chapter 10. A Bit More of Analytical Mechanics	16	10
CM TOTAL:	252	202

Additional file (available upon request):	*Pages*	*Problems*
Exercise and Test Problems with Model Solutions	338	202 + 45 = 247

EM: Classical Electrodynamics	Pages	Exercise Problems
Table of Contents	4	–
Chapter 1. Electric Charge Interaction	20	20
Chapter 2. Charges and Conductors	68	47
Chapter 3. Dipoles and Dielectrics	28	30
Chapter 4. DC Currents	16	15
Chapter 5. Magnetism	42	29
Chapter 6. Electromagnetism	38	31
Chapter 7. Electromagnetic Wave Propagation	70	43
Chapter 8. Radiation, Scattering, Interference, and Diffraction	38	28
Chapter 9. Special Relativity	56	42
Chapter 10. Radiation by Relativistic Charges	40	15
EM TOTAL:	420	300

Additional file (available upon request):	*Pages*	*Problems*
Exercise and Test Problems with Model Solutions	464	300 + 50 = 350

QM: Quantum Mechanics	*Pages*	*Exercise Problems*
Table of Contents	4	–
Chapter 1. Introduction	26	18
Chapter 2. 1D Wave Mechanics	76	47
Chapter 3. Higher Dimensionality Effects	64	49
Chapter 4. Bra-ket Formalism	52	36
Chapter 5. Some Exactly Solvable Problems	48	55
Chapter 6. Perturbative Approaches	36	31
Chapter 7. Open Quantum Systems	54	17
Chapter 8. Multiparticle Systems	52	35
Chapter 9. Introduction to Relativistic Quantum Mechanics	36	22
Chapter 10. Making Sense of Quantum Mechanics	16	1
QM TOTAL:	464	311

Additional file (available upon request):	*Pages*	*Problems*
Exercise and Test Problems with Model Solutions	574	311 + 69 = 380

SM: Statistical Mechanics	*Pages*	*Exercise Problems*
Table of Contents	4	–
Chapter 1. Review of Thermodynamics	24	18
Chapter 2. Principles of Physical Statistics	44	36
Chapter 3. Ideal and Not-So-Ideal Gases	34	30
Chapter 4. Phase Transitions	36	24
Chapter 5. Fluctuations	44	30
Chapter 6. Elements of Kinetics	38	18
SM TOTAL:	224	156

Additional file (available upon request):	*Pages*	*Problems*
Exercise and Test Problems with Model Solutions	272	156 + 26 = 182

Appendices	*Pages*
MA: Selected Mathematical Formulas	16
UCA: Selected Units and Constants	4

References	*Pages*
A partial list of books used at work on the series	2